



# Solver Horse Race

A collaborative project between ETH Zurich and OLZ&Partners

# Simple Optimization Problem:

Diet Problem:

- Buy your dinner
- Fulfil daily vitamin requirements
- Minimize total meal costs
- No negative values for meals

Prices /kg

Beef	3.19 \$
Chicken	2.59 \$
Fish	2.29 \$
Ham	2.89 \$

Daily vitamin cover /kg

Vitamins:	A	C	B1
Beef	60%	20%	10%
Chicken	8%	0%	20%
Fish	8%	10%	40%
Ham	40%	40%	32%

# Simple Optimization Problems

Minimize cost:

$$\text{cost} = 3.19X_{\text{Beef}} + 2.59X_{\text{Chk}} + 2.29X_{\text{Fish}} + 2.89X_{\text{Ham}}$$

Constraints:

Vitamin A

$$60X_{\text{Beef}} + 8X_{\text{Chk}} + 8X_{\text{Fish}} + 40X_{\text{Ham}} \geq 100$$

Vitamin C

...

Vitamin B

...

$$X_{\text{Beef}} \geq 0$$

$$X_{\text{Chk}} \geq 0$$

$$X_{\text{Fish}} \geq 0$$

$$X_{\text{Ham}} \geq 0$$

# General Optimization Problems

minimize<sub>x</sub>  $f(x)$

$$g_1(x) \leq C_1$$

$$g_2(x) = C_2$$

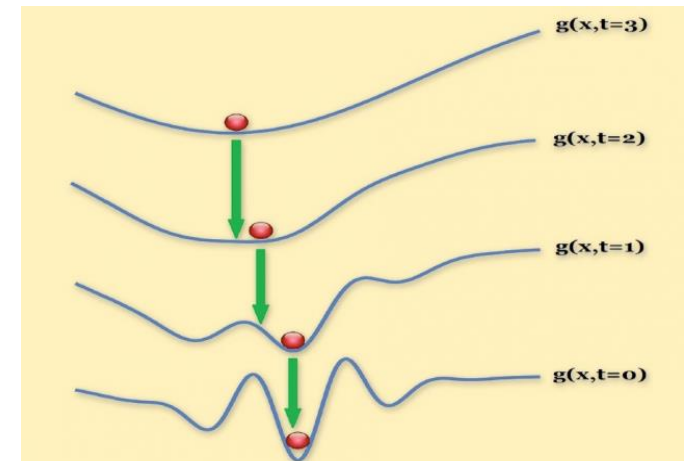
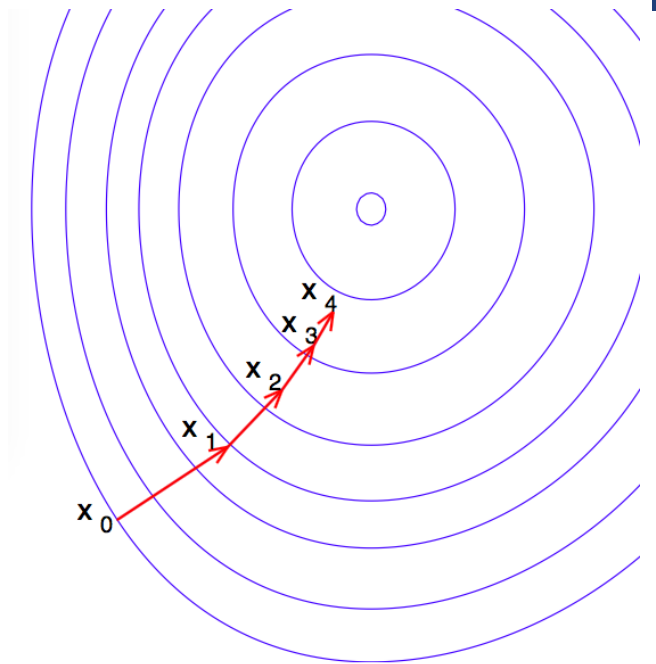
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Different problem types:

- Linear
- Quadratic
- Non-linear
- Combinatorial

**Solvers:**

- a program to apply an algorithm that finds optimal values of the variables.
- Many algorithms
  - Precision and run-time vary
- Solvers choose fastest and most reliable method



# What do we need solvers for?

Minimizing portfolio risk:

- Min-Variance Portfolio:

$$\begin{aligned} & \min_w w' \Sigma w \\ \text{s.t.} & \\ & \sum_i^N w_i = 1 \\ & w_i \geq 0 \end{aligned}$$

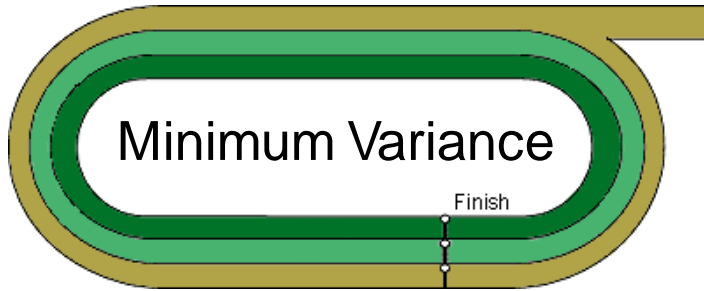
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- Add additional constraints to
  - decrease transaction costs
  - limit portfolio turnover
  - ....
- Problems often difficult to solve

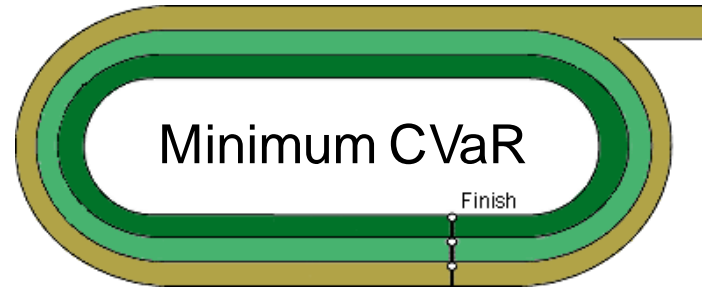
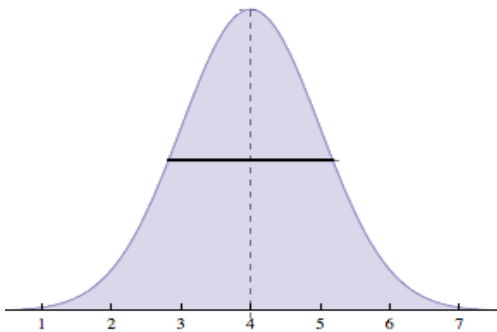
Which solver(s) fit the best to OLZ?

# Solver Horse Race

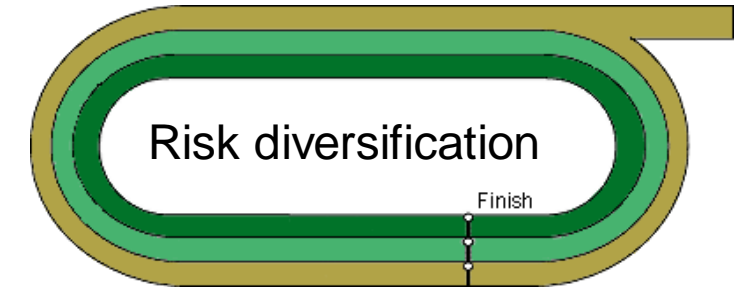
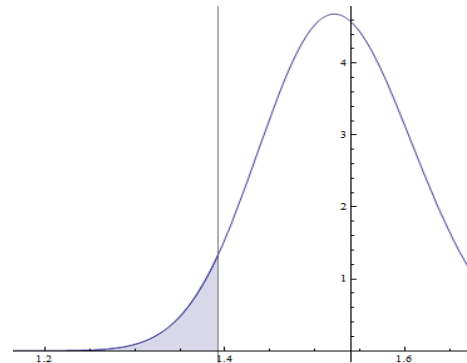
Three race tracks:



- Minimize portfolio volatility
- Quadratic problem



- Minimize Tail Risk
- Linear problem



- Minimize variance of risk contributions
- Non-linear problem

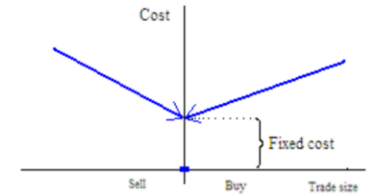
$$\min \left( R_i - \frac{1}{N} \cdot R \right)^2$$

# Solver Horse Race

Multiple races with different constraints:

- **Box/Group Constraints**  
Constrain the weight of individual assets or groups of assets
- **Turnover**  
Constrain the total change of your portfolio
- **Buy-In**  
Forbid unreasonably small trades

- **Transaction costs**  
Decrease your total trading costs  
Can be linear and/or fix per trade



- **Tail Dependence**  
Increase investment in uncorrelated assets
- **Effective number of bets**  
Diversify your investment

# Solver Horse Race

- Problem formulation and solver control often have completely different syntax

→ AMPL:

$$\begin{array}{l}
 \min_w \quad w' \Sigma w \\
 \text{s.t.} \\
 \sum_i^N w_i = 1 \\
 w_i \geq 0
 \end{array}
 \longrightarrow \text{Solver}$$

- Modeling language to unify problem formulation and solver control
- Fast problem switching
- Large solver support

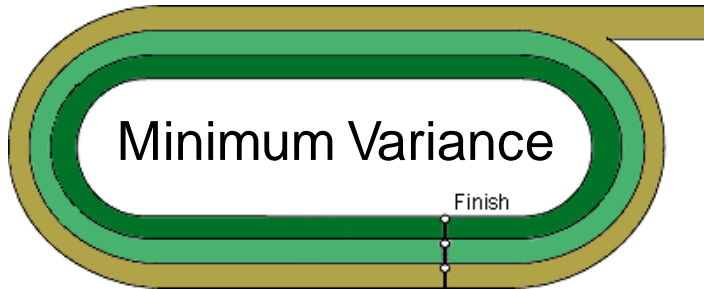
## Race Measurements:

- Computation time
- Precision
  - Risk minimization
  - Constraint fulfilment
- Test on a set of 1461 assets, data from OLZ



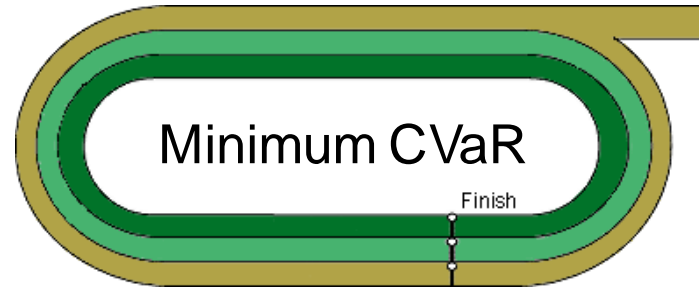
# Solver Horse Race

Three race tracks:



## Horses

- CPLEX (commercial)
- XPRESS (commercial)
- GUROBI (commercial)
- SNOPT (commercial)
- MINOS (commercial)
- CONOPT (commercial)
- KNITRO (commercial)
- LOQO (commercial)
- CBC (open source)



## Horses

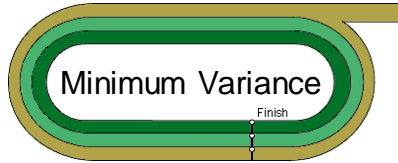
- CPLEX (commercial)
- XPRESS (commercial)
- GUROBI (commercial)
- SNOPT (commercial)
- MINOS (commercial)
- CONOPT (commercial)
- KNITRO (commercial)
- LOQO (commercial)
- CBC (open source)
- LPSOLVE (open source)



## Horses

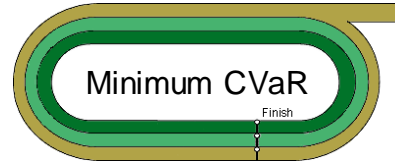
- IPOPT (open source)
- BONMIN (open source)
- COUENNE (open source)
- SNOPT (commercial)
- MINOS (commercial)
- CONOPT (commercial)
- KNITRO (commercial)
- LOQO (commercial)

# No Constraints - Computation Time



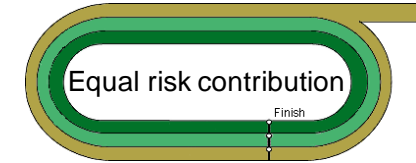
1.	XPRESS	6 s
2.	CPLEX	~10 s
	GUROBI	
	SNOPT	
5.	KNITRO	24 s
6.	CONOPT	~110 s
	LOQO	
8.	MINOS	190 s
9.	CBC	DNF

- CBC was not able to solve MV-problems (memory)



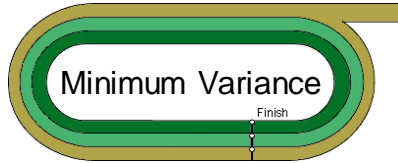
1.	CPLEX	~1 s
	XPRESS	
	GUROBI	
	CBC	
5.	SNOPT	~ 4 s
	MINOS	
7.	CONOPT	9 s
8.	LOQO	18 s
9.	KNITRO	44 s
10	LPSOLVE	DNF

- LPSOLVE was not able to solve any CVaR-problems (size)

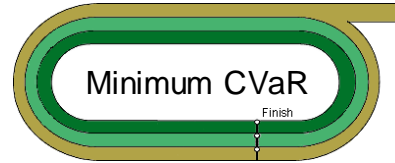


1.	SNOPT	25 s
2.	KNITRO	~48 s
	CONOPT	
4.	MINOS	72 s
5.	BONMIN	~ 140 s
	LOQO	
7.	IPOPT	200 s
8.	COUENNE	260 s

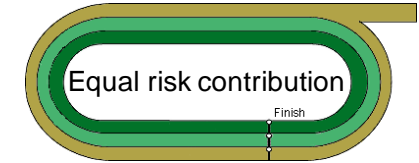
# No Constraints - Precision



1.	XPRESS	✓
	CPLEX	
	GUROBI	
	SNOPT	
	KNITRO	
	CONOPT	
	LOQO	
	MINOS	
9.	CBC	DNF

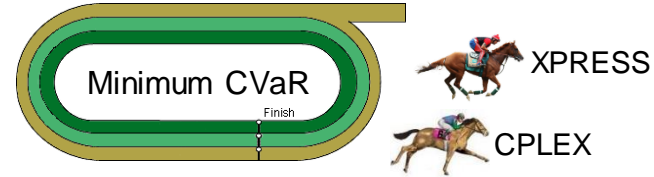
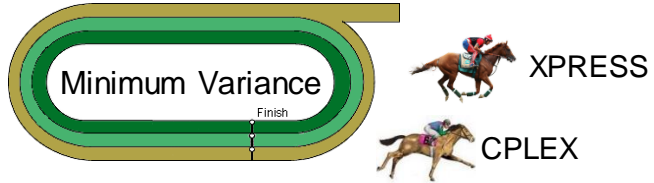


1.	CPLEX	✓
	XPRESS	
	GUROBI	
	KNITRO	
	SNOPT	
	MINOS	
	CONOPT	
	LOQO	
9.	CBC	Objective not converged
10	LPSOLVE	DNF



1.	SNOPT	✓
	KNITRO	
	CONOPT	
	MINOS	
	BONMIN	
	LOQO	
7.	IPOPT	Objective not converged
	MINOS	

# No Constraints - Result

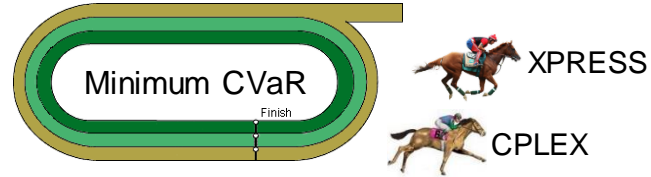
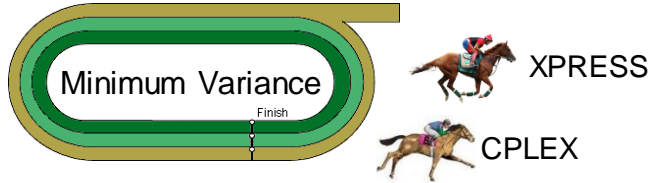


1.	XPRESS	6 s	✓
2.	CPLEX	~10 s	
	GUROBI		
	SNOPT		
5.	KNITRO	24 s	
6.	CONOPT	~110 s	
	LOQO		
8.	MINOS	190 s	
9.	CBC	DNF	

1.	XPRESS	~1 s	✓
	CPLEX		
	GUROBI		
4.	SNOPT	~ 4 s	
	MINOS		
6.	CONOPT	9 s	
7.	LOQO	18 s	
8.	KNITRO	44 s	
9.	CBC	~1 s	
10.	LPSOLVE	DNF	

1.	SNOPT	25 s	✓		
	2.			CONOPT	~48 s
				KNITRO	
4.	BONMIN	~ 140 s			
	LOQO				
6.	COUENNE	260 s			
7.	MINOS	72 s		Objective	
8.	IPOPT	200 s			

# Box/Group Constraints

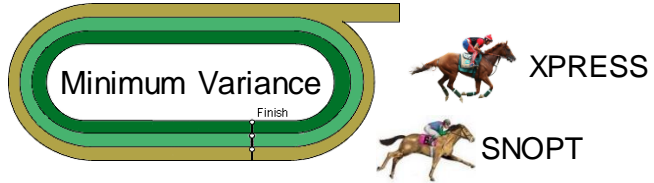


1.	XPRESS	6 s	Constr. violations
2.	CPLEX	~10 s	
	GUROBI		
	SNOPT		
5.	KNITRO	26 s	
6.	CONOPT	~53 s	
7.	LOQO	120	
8.	MINOS	150 s	

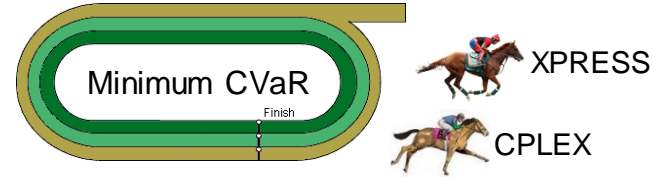
1.	XPRESS	~1 s	Constr. violations
	CPLEX		
	GUROBI		
4.	CONOPT	~ 5 s	Objective+ Constr. viol
5.	MINOS	9 s	
	SNOPT		
7.	LOQO	14 s	
8.	KNITRO	5 s	Constr. Violations
9.	CBC	160 s	

1.	SNOPT	15 s	✓
2.	BONMIN	~ 70 s	
	KINTRO		
4.	CONOPT	~ 150 s	
	LOQO		
6.	IPOPT	130	Objective+ Constr. viol
7.	MINOS	36 s	
8.	COUENNE	DNF	

# Turnover Constraints



1.	XPRESS	6 s	✓
	SNOPT	5 s	Constr. viol
3.	GUROBI	10 s	✓
4.	MINOS	9 s	Constr. viol
5.	CPLEX	16 s	✓
6.	CONOPT	~29 s	
	KNITRO		
8.	LOQO	410 s	

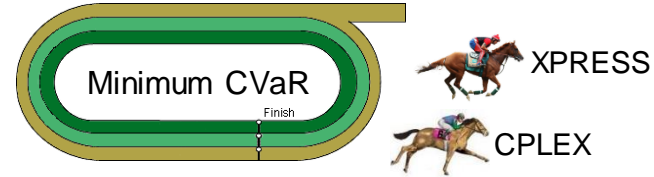
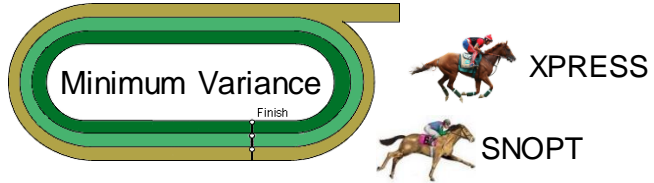


1.	XPRESS	~1.5 s	✓
	CPLEX		
	GUROBI		
4.	CBC		Constr.viol
5.	SNOPT	~6 s	Constr.viol
	MINOS		
7.	CONOPT	12 s	
8.	LOQO	22	
9.	KNITRO	17 s	Objective



1.	SNOPT	7 s	Constr.viol
2.	MINOS	15 s	
3.	CONOPT	34 s	✓
4.	KNITRO	80 s	Constr.viol
5.	BONMIN	~200 s	
	IPOPT		
7.	LOQO	DNF	
8.	COUENNE	DNF	

# Linear Transaction Costs

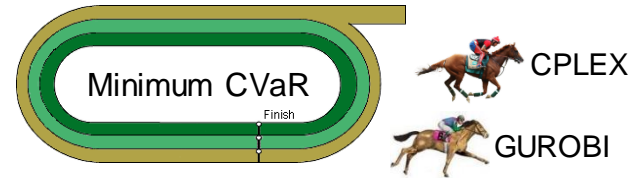
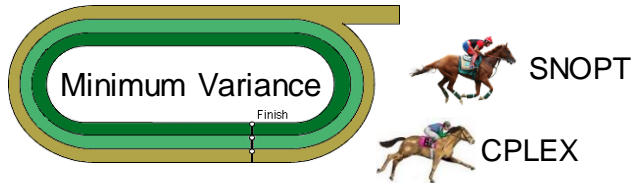


1.	SNOPT	5 s	✓
	XPRESS	7 s	
3.	GUROBI	10s	Constr. viol
	MINOS	8 s	
5.	CPLEX	~20 s	✓
	CONOPT		Constr. viol
7.	KNITRO	35 s	
8.	LOQO	400 s	✓

1.	XPRESS	~1.5 s	✓
	GUROBI		
	CPLEX		
4.	MINOS	5 s	Constr.viol
	SNOPT		
6.	CONOPT	10 s	
7.	LOQO	25 s	✓
8.	KNITRO	20 s	Objective
9.	CBC	5 s	Objective

1.	SNOPT	7 s	✓
2.	MINOS	19 s	Constr.viol
3.	CONOPT	38 s	✓
4.	KNITRO	130 s	Constr. Viol
5.	BONMIN	~300 s	Constr.viol
	IPOPT		
7.	LOQO	800 s	✓
8.	COUENNE	DNF	

# Tail Dependence Constraints



1.	SNOPT	29 s	✓
2.	CPLEX	~50 s	
	GUROBI		
4.	KNITRO	76	
5.	CONOPT	380 s	
6.	MINOS	850 s	
7.	XPRESS	15 s	Ignored constraints
	LOQO	DNF	

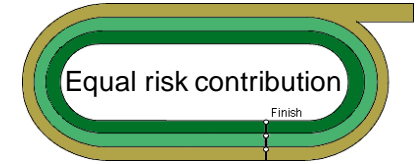
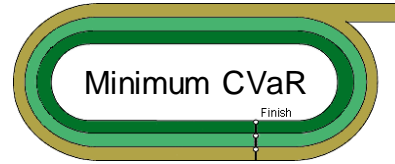
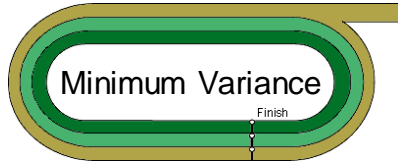
1.	GUROBI	~20 s	✓
	CPLEX		
	SNOPT		Constr.viol
4.	KNITRO	100 s	✓
5.	MINOS	290 s	Constr.viol
6.	XPRESS	15 s	Ignored constraints
	CONOPT	DNF	
	LOQO		
	CBC		

1.	SNOPT	29	✓
2.	CONOPT	~150 s	
	KNITRO		
4.	MINOS	~300 s	
	BONMIN		
	IPOPT		
7.	LOQO	DNF	
	COUENNE		

- Additional box/group constraints help XPRESS



# Effective Number of Bets Constraint



- Heavily non-linear function

$$\text{ENB} = \exp\left(-\sum_i^N p_i \ln p_i\right)$$

$$p_i = \frac{(Tw)_i \cdot (C \cdot Tw)_i}{\sum_i^N (Tw)_i \cdot (C \cdot Tw)_i}$$

- All non-linear solvers broke down at ~80 assets

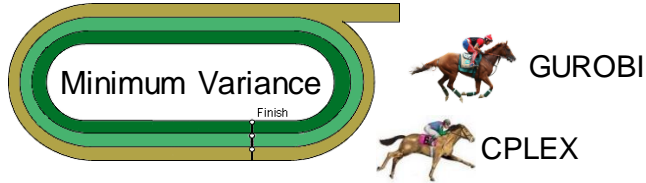
→ Limit of numerical minimization

## Special races:

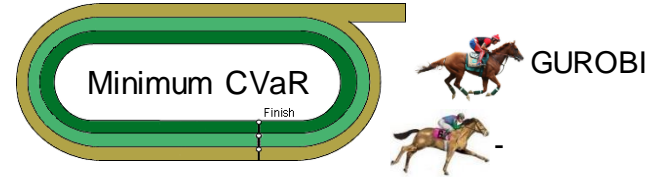
### Mixed integer problems

- Involve combinatorial computations
- Many solvers are not capable of solving mixed-integer problems

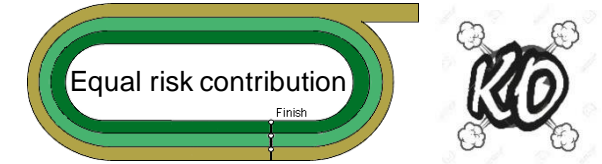
# Buy-In Constraints



1.	GUROBI	6 s	✓
2.	CPLEX	33 s	
	KNITRO	43 s	
4.	XPRESS	11 s	Constraints ignored
	CBC	DNF	



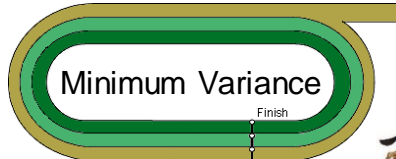
1.	XPRESS	1 s	✓
2.	CPLEX	2 s	
	GUROBI		
4.	KNITRO	58 s	Constraints ignored
	CBC	9.32	



1.	BONMIN	DNF	
	KNITRO		
	COUENNE		

- Again, Box/Group constraints help XPRESS
- CPLEX and XPRESS often completely ignore Buy-In constraints when combining with other constraints!

# Fixed & Linear Transaction Costs

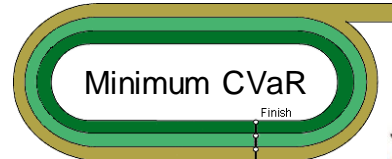


Minimum Variance  
Finish

GUROBI

CPLEX

1.	GUROBI	22 s	Const. viol
2.	CPLEX	69 s	✓
3.	XPRESS	10 s	Objective+ Constr. viol
4.	KNITRO	DNF	
	CBC		

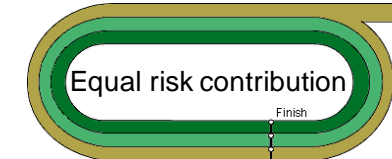


Minimum CVaR  
Finish

GUROBI

CPLEX

1.	CPLEX	23 s	Const. viol
	GUROBI	33 s	
3.	XPRESS	2 s	Objective+ Constr. viol
	CBC	40 s	
5.	KNITRO	DNF	



Equal risk contribution  
Finish

KO

1.	BONMIN	DNF	
	KNITRO		
	COUENNE		

# Who won?

Without Mixed-Integer Problems:

## SNOPT

- Always found optimal solution
- Constraints always fulfilled (in soft sense)
- Fastest for both linear & non-linear problems

Runner up:

- CPLEX/GUROBI for Min-Var and Min-CVaR
- CONOPT for non-linear problems

With Mixed-Integer Problems:

## GUROBI

- Always found optimal solution
- Constraints always fulfilled (in soft sense)
- Faster than CPLEX, more reliable than other solvers

Runner up:

- CPLEX

**Thank you for your attention**

